		<u>compiled by , Musicia As</u>	<u> </u>
40. Probability	•	calculate the probability of a single event as either a fraction or a decimal	Probabilities should not be given as ratios. Problems could be set involving extracting information from tables or graphs.
	•	understand that the probability of an event occurring = 1 - the probability of the event not occurring	e.g. P(blue) = 0.8, find P(not blue)
	•	understand relative frequency as an estimate of probability	e.g. use results of experiments with a spinner to estimate the probability of a given outcome e.g. use probability to estimate from a
		calculate the probability of simple	population In possibility diagrams outcomes will
		combined events using possibility diagrams and tree diagrams where appropriate	be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the

branches.

For understanding

https://www.youtube.com/watch?v= 2ZXGFEXLIg

https://www.youtube.com/watch?v=PGA7PCs2tFl

https://www.youtube.com/watch?v=uzkc-qNVoOk

Jim plays for his local football team.

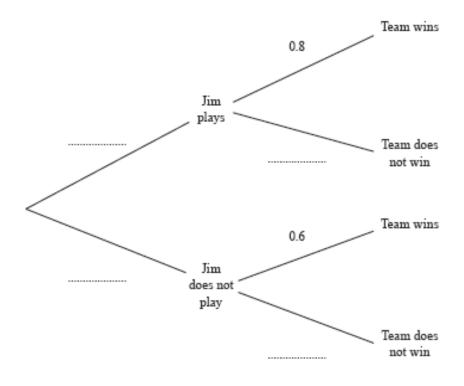
M/J19/11/Q18

The probability that Jim plays in the next match is 0.7.

If Jim plays in the match, the probability of his team winning is 0.8.

If Jim does not play in the match, the probability of his team winning is 0.6.

(a) Complete the tree diagram.



(b) Calculate the probability that Jim's team wins their next match.

.....[2]

[2]

2









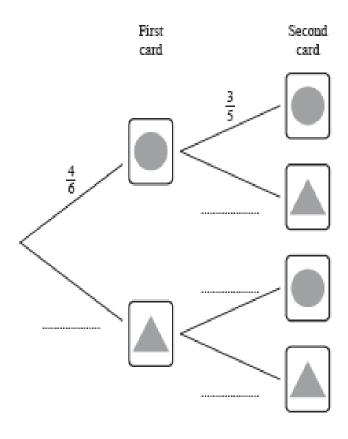




Nima has these six cards. Each card has a shape on it. She takes two cards at random without replacement.

M/J19/12/Q17

(a) Complete the tree diagram.



[2]

(b) Find the probability that the shapes on Nima's two cards are the same. Give your answer as a fraction.

3 A machine puts beads of different colours and sizes into packets.

The beads are selected at random from a large container and the selection of each bead for a packet is independent of all others.

The table shows information on the contents of six packets.

Packet	1	2	3	4	5	6	Tota1
Total number of beads	15	14	19	18	16	18	100
Number of blue beads	6	5	8	6	8	7	

									1
Nı	umber of blue beads	6	5	8	6	8	7		
(a)	Calculate the relativ	e frequency	of the mad	hine selecti	ng a blue b	ead.			
					Answe	r			[1]
(b)	Calculate how many	blue beads	you would	expect in a	packet of 3	0 beads.			
					Answe	r			[1]
(c)	The probability that	the machin	e selects a r	ed bead is ().17.				
	Calculate the probab	oility that th	e machine o	does not sel	ect a red be	ad.			
					Answe	2.			r11
					22112770				[-]
	e-sided spinner is nu		2, 3, 4 and	5.	N18/11/15	:			
	rraf spun the spinner results are shown in			O,	1410/11/15				
	27 1 1							Т	
	Number spinne	er lands on	1	2	3	4	5	-	
	Frequency		30	25	50	55	40]	
(a)	Calculate the relativ	ve frequenc	y that the	spinner lan	ds on 3.				
					4				F17
					Ansv	ver			[1]
(b)	Meriam spins the sp	pinner 20 ti	mes.						
	How many times w	ould you e	xpect the sp	pinner to la	nd on 3?				
					4				F17
						ver			[1]
(c)	Ashraf claims: "My Is his claim correct"	7 results sho ?	ow that the	spinner is	fair".				
	Give a reason for y	our answer							
	because								

5

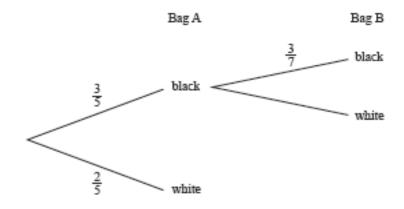


Bag A contains 3 black and 2 white beads. Bag B contains 2 black and 4 white beads. O/N18/12/17

Bag B

A bead is chosen, at random, from Bag A and placed in Bag B. A bead is then chosen, at random, from Bag B.

(a) Complete the tree diagram.



(b) Find the probability that a black bead is taken from Bag B.

Answer[2]

[2]

6	Basia records the colour of 100 cars passing the school gate.	
	Her results are recorded in the table.	M/J18/11/12

Colour of car	Black	Grey	Red	Blue	Other
Frequency	43	18	12	9	18

	Colour of car	Black	Grey	Red	Blue	Other	
	Frequency	43	18	12	9	18	
(a)	Use Basia's result	s to estimate th	ne probability (that the next ca	ar seen is a blu	ue car.	
				An	swer		[1]
(b)	In the next hour, 5	00 cars pass th	ne school gate.				
	Use Basia's result	s to estimate th	ne number of t	hese cars that	are red.		
				An	swer		[1]
(c)	Colin records the His results are sho			assing the scho	ool gate.		
	Colour of car	Black	Grey	Red	Blue	Other	
					20		ı
	Frequency	34	10	18	28	10	
	Frequency Use Basia's and C when 500 cars pas	colin's combin	ed results to e				d be seer
	Use Basia's and C	colin's combin	ed results to e	stimate the nu		cars that woul	
(d)	Use Basia's and C	Colin's combin s the school g	ed results to e ate.	stimate the nu	smber of red o	cars that woul	
(d)	Use Basia's and C when 500 cars pas	Colin's combin is the school g nates in part (ed results to e ate. b) or in part (stimate the nu An c) is likely to t	swer	cars that woul	[1]

7 A dice is thrown 400 times.

M/J18/12/12

The results are shown in the table.

(a) Find the relative frequency of throwing the number 2.

Number thrown	1	2	3	4	5	6
Frequency	65	80	70	75	50	60

	Answer	[1]
(b)	Imran throws the dice 1000 times.	
	How many times would you expect the number 2 to be thrown?	
	Answer	[1]

8

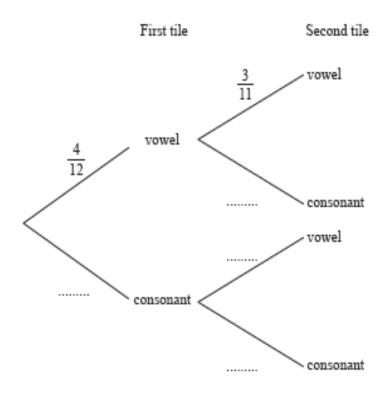
Y G 0 Ν 0 М Ε Т R Twelve lettered tiles spelling the word TRIGONOMETRY are placed inside a bag. (a) A tile is taken at random from the bag. M/J18/21/4 Find the probability that the tile shows a letter R. Give your answer as a fraction in its simplest form. Answer[1] (b) All the tiles are placed back in the bag, a tile is then taken at random and placed on the table. A second tile is taken at random and placed to the right of the first tile. A third tile is taken at random and placed to the right of the second tile. 1st 2nd 3rd Find the probability that, in the order the tiles were placed on the table, they spell GET.

Answer[2]

- (c) Vowels are the letters A, E, I, O and U.
 - All other letters are consonants.

All the twelve tiles are placed back in the bag and two tiles are taken at random, without replacement.

(i) Complete the tree diagram.



(ii) Find the probability that the tiles both show vowels.

Answer[1]

[2]

(iii) Find the probability that one tile shows a vowel and one tile shows a consonant.

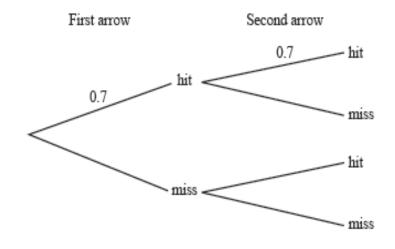
Answer[2]

9 The 262 students at a college each study one of the languages shown in the table.

	French	German	Spanish	Italian	Japanese	Tota1
Boys	27		48	19		123
Girls		32	54		12	
Tota1		53		30		262

a)	Con	uplete the table.	[3]
b)	Find	the probability that	
	(i)	a girl, chosen at random, studies Spanish,	
			 [1]
	(ii)	a boy, chosen at random, studies French or Italian,	
			 [1]
	(iii)	a student, chosen at random, does not study German.	
			 Г11

- 10 Each time an archer fires an arrow, the probability that she hits the target is 0.7.
 She fires two arrows.
 - (a) Complete the tree diagram.



[1]

- (b) Find the probability that
 - (i) she hits the target twice,

Answer	[1]	l

(ii) she hits the target exactly once.

Answer[1]

11	Adam	has a bag containing 9 balls, numbered from 1 to	O/N17/21/4	
(a) Ada	am takes a ball at random from the bag and replaces it.		
	Fin	d the probability that the ball has an odd number.		
(b		am takes a ball from the 9 balls in the bag, notes the most then takes a second ball from the bag, notes the number Work out the probability that both numbers are odd.		[1]
	(ii)	Work out the probability that one number is odd and	Answer[the other is even.	[1]
(c) Ada	am now takes two balls from the 9 balls in the bag, wit	Answer[[2]
	Wo	ork out the probability that the two numbers are either b	ooth odd or both even.	
			Answer[[3]

A bag contains red and blue pegs.

There are 40 pegs in the bag.

The probability of choosing a red peg from the bag is 0.4.

(a) Work out the number of red pegs in the bag.

Answer[1]

(b) More red pegs are added to the bag.

Work out the number of red pegs that must be added to the bag so that the probability of choosing a blue peg is 0.2.

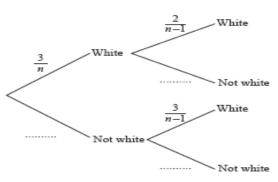
Answer[2]

A bag contains n balls. M/J17/12/24 3 of the balls are white. Two balls are taken from the bag, at random, without replacement.

(a) Complete the tree diagram.

First ball

Second ball



[2]

(b) The probability that both balls are white is \(\frac{1}{15}\).

Show that $n^2 - n - 90 = 0$.

[2]

(c) Find the value of n.

14 Rowena spins two fair spinners, each numbered 1 to 4.

Answer

Her score is the value when the numbers on the two spinners are multiplied together. The table shows some of Rowena's possible scores.

M/J17/22/3

×	1	2	3	4
1	1	2	3	4
2	2	4		
3				
4				

	-
(a)	Complete the table of possible scores. [2]
(b)	Find the probability that Rowena's score is less than 4.
	Answer[1]
(c)	Find the probability that Rowena's score is an even number.
(0)	Give your answer as a fraction in its lowest terms.
	Answer[2]
(d)	Phoebe says that Rowena's score is more likely to be a square number than a factor of 6.
	Is she correct? Show your working.

15 2 3 O/N16/1218

Four cards are marked with the numbers 1, 2, 3 and 4.

One card is chosen at random.

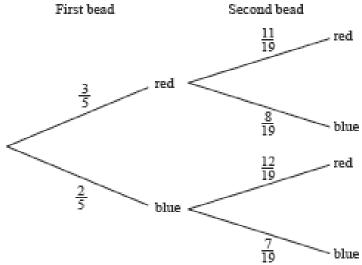
A second card is then chosen, at random, from the remaining three cards. The sum of the numbers on the two chosen cards is calculated.

(a) Complete the table to show the possible outcomes.

	First card							
rd		1	2	3	4			
	1							
Second card	2							
Sec	3							
	4							

[1] (b) What is the probability that the sum is less than 2?[1] (c) What is the probability that the sum is greater than 5?

16 A bag contains R red beads and B blue beads. O/N16/21/10(b) Two beads are chosen, at random, without replacement. The tree diagram shows the possible outcomes and their probabilities.



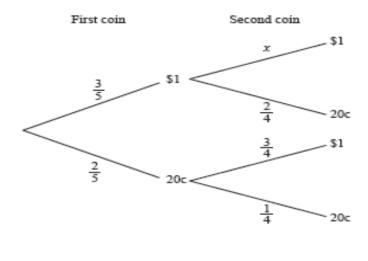
(ī)	Calculate the probability that both beads are red.
(ii)	Answer
(iii)	Answer
(iv)	Answer
	Calculate the probability that, of the two chosen beads, neither has a yellow spot.

Answer[2]

17 Amira has three \$1 coins and two 20c coins in her purse. She picks out coins at random, one after the other. The coins are not replaced.

The tree diagram shows the possible outcomes and their probabilities when picking out two coins.

O/N16/22/11(b)



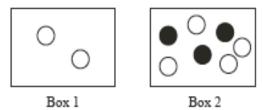
(i)	Find x.
	Answer[1]
(ii)	Find the probability that the total value of the two coins picked out is 40 cents.
	Answer[1]
(iii)	Find the probability that the total value of the two coins picked out is $$1.20$.

(iv) At a car park, the charge is \$1.40.
Amira picks out three coins, one after the other.

Find the probability that the total value of the three coins is \$1.40.

Answer		[2]
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Answer[2]



Вох	1 contains 2 white balls. Box 2 contains 4 white balls an	d 3 black balls.
(a)	Ann chooses, at random, one ball from each box.	16/116/11/006
	(i) Find the probability that these balls are both black.	M/J16/11/Q26
		Answer[1]
	(ii) Find the probability that these balls have different of	colours.
		Answer[1]
(b)	From the original contents of Box 2, Belle chooses, at ra	ndom, two balls without replacement.
	Find the probability that these balls are both white.	
		Answer[1]
(c)	Carla chooses one of the boxes at random. With the original box contents, she then chooses, at rand	lom, one ball from this box.
	Find the probability that the ball is white.	

Answer[2]	Answer		[2]	
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M/J16/12/20

- 19 A bag contains 10 counters of which 8 are blue and 2 are white. Two counters are taken from the bag at random without replacement.
 - (a) Complete the tree diagram to show the possible outcomes and their probabilities.

Second counter

Blue

8 10

White

White

White

[1]

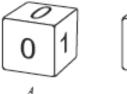
- (b) Find, as a fraction, the probability that
 - (i) both counters are blue,

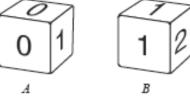
First counter

Answer[1]

(ii) one counter is blue and the other is white.

Answer[2]





	22	,
The The	e diagram shows two fair dice. e numbers on dice A are 0, 0, 1, 1, 1, 3. e numbers on dice B are 1, 1, 2, 2, 2, 3. en a dice is rolled, the score is the number on the top face	ž.
(a)	Dice A is rolled once.	
	Find the probability that the score is not 3.	
		[1]
(b)	Dice A is rolled twice.	
	Find the probability that the score is 0 both times.	
		[2]
(c)	Dice A is rolled 60 times.	
	Calculate an estimate of the number of times the score	is 0.
		[1]

15

(d)	Dice A and dice B are each rolled once.
	The product of the scores is recorded.

(i) Complete the possibility diagram.

	3	0	0				
Dice B	2	0	0				
	2	0	0				
Dicco	2	0	0				
	1	0	0				
	1	0	0	1	1	1	3
	_	0	0	1	1	1	3
				Die	e.A		

ı	II.

(ii) Find the probability that the product of the scores is

(a) 2,

......[1]

(b) greater than 3.

.....[1]

(e) Eva keeps rolling dice B until 1 is scored.

Find the probability that this happens on the 5th roll.

21

Humanities:	Geography (G)	
	History (H) Religious studies (R)	

Science: Physics (P)
Chemistry (C)
Biology (B)

A student has to choose one humanities subject and two different science subjects.

(a) Complete the table to show the possible outcomes.

M/J17/21/3

Answer	Humanities	Science
	G	P and C
	G	P and B

		[2]
(b)	Kha	dif chooses his subjects at random.
	(i)	Find the probability that he chooses Geography.
		Answer[1]
	(ii)	Find the probability that he chooses Physics.
		Answer[1]
	(iii)	Find the probability that he chooses both Religious studies and Chemistry.
		Answer[1]

Mark Scheme

1(a)	[0].7, [0].3, [0].2, [0].4 correctly placed	2	M1 for two or three correct
1(b)	[0].74 oe	2	M1 for 0.7×0.8 or 0.3×0.6 oe

2(a)	$\frac{2}{6}$ on first branch $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{5}$ on second set	2	B1 for two or three completed correctly
2(b)	$\frac{14}{30}$ oe	2	M1 for $\frac{4}{6} \times \frac{3}{5}$ oe or their $\frac{2}{6} \times their \frac{1}{5}$ oe

Question	Answer	Marks	Part marks
3(a)	0.4 oe	1	
3(b)	12 or their (a) × 30 ft	1	
3(c)	0.83	1	

4(a)	$\frac{50}{200}$ oe	1	
4(b)	5	1	FT $20 \times their$ (a) provided their (a) < 1.
4(c)	No, with a supporting reason	1	e.g.: [has been spun enough times for results to be reliable.]; results are significantly different from those for a fair spinner.

5(a)	$\frac{4}{7}$	1	
	$\frac{2}{7}$ (black) and $\frac{5}{7}$ (white) with two branches and both labels	1	
5(b)	$\frac{13}{35}$ oe	2	FT $\frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times (their \frac{2}{7})$ or M1 for $\frac{3}{5} \times \frac{3}{7}$; or for $\frac{2}{5} \times (their \frac{2}{7})$

6(a)	$\frac{9}{100}$ oe	1	
6(b)	60	1	
6(c)	75	1	
6(d)	(c) because based on a larger sample oe	1	

7(a)	$\frac{80}{400}$ oe	1	
7(b)	200	1	FT (their(a)) \times 1000 where 0 < their (a) < 1

8(a)	$\frac{1}{6}$ cao	1	
8(b)	1/660 oe	2	M1 for $\frac{1}{12} \times \frac{1}{11} \times \frac{2}{10}$ oe or SC1 for $\frac{1}{12} \times \frac{1}{12} \times \frac{2}{12}$ or answer $\frac{1}{864}$ or $\frac{1}{12}, \frac{1}{11}, \frac{2}{10}$
8(c)(i)	$\frac{8}{12}$, $\frac{8}{11}$, $\frac{4}{11}$, $\frac{7}{11}$ oe correctly placed	2	B1 for two correct
8(c)(ii)	$\frac{1}{11}$ oe	1	
8(c)(iii)	$\frac{16}{33}$ oe	2	M1 for $\frac{4}{12} \times \frac{8}{11}$ or $\frac{8}{12} \times \frac{4}{11}$ oe

	_			_	7011	ipit	cu	U y	, 1*1	usic	<u> </u>	<u> </u>	
9(a)		F	G	S	I	J	Tot	t		3	B2	for 6 or	7 correct
	В		21			8					or I	B1 for 3	, 4 or 5 correct
	G	30		102	11	20	139)					
	Tot	57		102		20		_					
9b)(i)	$\frac{54}{their}$		oe isv	N						1	FT	their ta	ble
9(b)(ii)	46 123	oe isv	N							1			
9(b)(iii)	$\frac{209}{262}$	oe is	w							1			
10(a)	Probal			and 0.	3 on t	he			1				
10(b)(i)	0.49 o	e							1				
10(b)(ii)	0.42 o	e							1				gram, provided their diagram ess than 1, and 0 < ans. < 1.
	1										1		
11(a)	$\frac{5}{9}$ oe									1			
11(b)(i)	$\frac{25}{81}$ oe									1			
11(b)(ii)	$\frac{40}{81}$ oe									2	M1	for the	$\frac{ir5}{9} \times \frac{(9-their5)}{9}$ soi or
	81										1		9
											- ($\frac{2ir5}{9} \times \frac{4}{9}$	
11(c)	$\frac{4}{9}$ oe 1	ıfww								3	M2	for $\frac{5}{9}$ ×	$\frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$
											or N	M1 for	$\frac{4}{9} \times \frac{3}{8} \text{ or } \frac{5}{9} \times \frac{4}{8}$
12(a)	16											1	
12b)	80											2	B1 for 120 or 96 seen
													or M1 for $\frac{24}{40+x} = \frac{1}{5}$ or $\frac{16+x}{40+x} = \frac{4}{5}$ oe

$\frac{n-4}{n-1}$ oe		
13(b) $\frac{3}{n} \times \frac{2}{n-1} = \frac{1}{15}$ Correct rearrangement with at least	M1	
one further step to reach $n^2 - n - 90 = 0$		
13(c) 10	2	B1 for solutions 10, -9 seen or M1 for $(n - 10)(n + 9)$ [= 0] or for $\frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times -90}}{2 \times 1}$ or better

14(a)	6 8 3 6 9 12 4 8 12 16	2	B1 for at least 6 correct
14(b)	5/16 or 0.3125 or 31.25%	1	FT <i>their</i> complete table (decimals or percentages correct to at least 3sf)

14(c)	$\frac{3}{4}$ cao	2	B1 for $\frac{12}{16}$ or $\frac{6}{8}$ or $\frac{their12}{16}$ oe
14(d)	No with square 6 and factors 7 seen or square $\frac{6}{16}$ and factors $\frac{7}{16}$ seen or $1.4.4.4.9$ 16 and $1.2.2.3$ 3.6 6 seen or $1^2 2^2 2^2 2^2 3^2 4^2$ and $1.2.2.3$ 3.6 6 seen		B1 for square $\frac{6}{16}$ or factors $\frac{7}{16}$ or 1444916 seen or $1^2 2^2 2^2 2^2 3^2 4^2$ seen or 1223366 seen or square 6 and factors 7

15 (a)	_ 3 4 5	
	3 - 5 6	
	4 5 - 7	
	5 6 7 –	
(b)	0 1	
(c)	$\frac{4}{12}$ oe; or FT their table	

	oompica by , Mastara Kon					
16 (i)	$\frac{33}{95}$ oe	1				
(ii)	$\frac{48}{95}$ oe	2	M1 for $\frac{3}{5} \times \frac{8}{19} + \frac{2}{5} \times \frac{12}{19}$ Or SC1 for answer $\frac{24}{95}$			
(iii)	12 cao	1				
(iv)	$\frac{91}{190}$ oe	2	M1 for $\frac{k}{n} \times \frac{k-1}{n-1}$ where $n > k > 1$			

17 (i)	$\frac{2}{4}$ oe	1	
(ii)	$\frac{2}{20}$ oe	1	
(iii)	$\frac{12}{20}$ oe	2	B1 for $\frac{3}{5} \times \frac{2}{4}$ or $\frac{2}{5} \times \frac{3}{4}$ seen
(iv)	$\frac{18}{60}$ oe	2	B1 for any correct sequence of three coins, $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \text{ or } \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} \text{ or } \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}$

18 (a) (i)	0	1	
(ii)	$\frac{3}{7}$	1	
(b)	$\frac{2}{7}$ oe	1	
(c)	$\frac{11}{14}$ oe	2*	M1 for $\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{4}{7}$

19 (a)	$\frac{2}{10}, \frac{2}{9}, \frac{8}{9}, \frac{1}{9}$ correctly positioned	1	
(b) (i)	$\frac{56}{90}$ oe	1*	
(ii)	$\frac{32}{90}$ oe	2ft*	M1 for $\frac{8}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{8}{9}$ ft <i>their</i> tree diagram with fractions < 1

20(a)	$\frac{5}{6}$	1	
20(b)	$\frac{4}{36}$ oe	2	M1 for $\frac{2}{6} \times \frac{2}{6}$
20(c)	20	1	

20(d)(i)	Diagram completed correctly	2	B1 for 3 correct columns or for 4 correct rows
	x x 3 3 3 9 x x 2 2 2 6 x x 2 2 2 6 x x 2 2 2 6 x x 1 1 1 3		
20(d)(ii)(a)		1FT	FT their (d)(i)
20(d)(ii)(b)	$\frac{4}{36}$ oe	1FT	FT their (d)(i)
20(e)	512 7776 oe	2	M1 for $\left(\frac{4}{6}\right)^k \times \frac{2}{6}$ oe $k = 3, 4$ or 5 only

21(a)	GCB, HPC, HPB, HCB, RPC,RPB, RCB	2	B1 for 5 correct and none incorrect or for 6 correct
21(b)(i)	$\frac{3}{9}$ or $\frac{1}{3}$ or 0.333() or 33.3()%	1	FT dep on B1 scored in (a)
21(b)(ii)	$\frac{6}{9}$ or $\frac{2}{3}$ or 0.666 – 0.667 or 66.6% – 66.7%	1	FT dep on B1 scored in (a)
21(b)(iii)	² / ₉ or 0.222() or 22.2()%	1	FT dep on B1 scored in (a) After 0 scored in (i) (ii) and (iii), SC1 for $\frac{3}{k}, \frac{6}{k}, \frac{2}{k}$